



Collective lattice resonances: Plasmonics and beyond

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ARTICLE INFO

Keywords:

Collective lattice resonance
Surface lattice resonance
Plasmonics
All-dielectric
Mie resonance
Localized surface plasmon resonance
Rayleigh anomaly

ABSTRACT

Engineering nanostructures with exceptionally high-Q resonances mediated by the Fano-type hybridization between discrete states associated with the periodicity of the structure and broadband resonances excited on constituent scatterers are the emerging field in optics and photonics. These collective lattice resonances (CLRs) attracted a lot of attention in recent years due to a number of their exciting applications in sensing, optical filtering, structural color printing, fluorescence enhancement, nanoscale lasing, and nonlinear optics, which resulted in a rapidly growing number of fundamental and experimental studies. CLRs have been discovered for arrays of plasmonic metal nanoparticles with strong electric dipole resonances nearly four decades ago. Thereafter, the scope of CLRs has gradually extended to all-dielectric and magneto-optical nanoparticles, 2D materials and other types of constituents, which has broadened the range of CLRs applicability and enriched their properties. We provide a comprehensive review of the recent progress in this field with a special emphasis on advances far beyond plasmonics.

1. Introduction

Four decades ago [1], quite simple yet insightful question has been asked: is it possible to arrange plasmonic nanoparticles (NPs) in such a way that the optical response of a cluster as a whole would have higher quality factor than the localized surface plasmon resonance of its constituents? It occurred, that being arranged in a regular 1D or 2D lattice with a period comparable with the wavelength of the incident illumination, arrays of plasmonic spheres may exhibit high-Q collective response [2,3]. These diffracting lattice modes are generally observed at wavelengths close to a Rayleigh anomaly (RA) of the array [4]. Almost two decades later, this phenomenon has been “rediscovered” for 1D chains of Ag nanoparticles (NPs) [5–7], 1D Au gratings [8–10], 2D arrays of nanoholes in perfect electric conductor thin films [11] and 3D periodic assemblies of metallic nanoshells [12].

In 2005, attempts to observe these collective oscillations in experiments with 1D arrays of Ag nanocylinders [13] pointed to the importance of coupling between NPs which immediately implied the necessity of large-area illumination. Three years later, in 2008, several groups independently have reported experimental evidence of high-Q collective response of 2D arrays of Au NPs [14–16], and, one year later, for Ag nanorods [17], with an explicit proof of the *surface* nature of these modes after careful measurement of the dispersion relations [18]. Since initially these collective lattice resonances (CLRs) have been demonstrated for plasmonic systems supporting localized *surface* plasmon resonances, they are widely referred to as *surface lattice resonances*. Progress in nanofabrication techniques allows to tailor a quality factor of CLRs up to 3 orders of magnitude larger than a localized resonance of an isolated constituent NP (Q-factors of 25 [14], 40 [15], 160 [18], 430 [19] and 2340 [20] are measured experimentally for 2D arrays of Au NPs, compare with ≈ 5 –15 Q-factor of localized plasmon resonance of a single Au NP). Recent reviews [21–26] and tutorials [27–29]

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on CLRs are focused on plasmonic systems, however, similar collective lattice modes can be observed in regular arrays of atoms [30], all-dielectric NPs [31] and transdimensional systems [32]. The variability of shape, composition and arrangement of the constituents (Fig. 1) gives rise to a number of exciting applications of CLRs in biosensing, vibrational spectroscopy, optical filtering, color printing, fluorescence enhancement, nanoscale lasing, and nonlinear optics. In this review, we provide a comprehensive discussion of CLRs, and overview recent progress in this field, including plasmonic, all-dielectric, transdimensional and atomically thin structures.

2. Fundamentals

2.1. Theory

The emergence of CLRs can be understood within the framework of the coupled dipole approximation (DA) [33]. The original formulation of DA is based on 1D chain of electric dipoles (EDs) [34], which has been further extended to a system of coupled EDs and magnetic dipoles (MDs) [35]. Following the latter approach, let us consider the array of N identical dipole scatterers embedded in a homogeneous medium and illuminated by a plane wave with $\mathbf{E}_{\text{inc}}(\mathbf{r}) = \mathbf{E}_0 \exp(i\mathbf{k} \cdot \mathbf{r})$ and $\mathbf{H}_{\text{inc}}(\mathbf{r}) = \mathbf{H}_0 \exp(i\mathbf{k} \cdot \mathbf{r})$, where \mathbf{E}_0 and \mathbf{H}_0 are amplitudes of the electric and magnetic fields, \mathbf{k} is a wave vector, and the time dependence $\exp(-i\omega t)$ is assumed and suppressed. The electric and magnetic dipole moments \mathbf{d}_i and \mathbf{m}_i induced on i th scatterer are coupled to dipoles on other $j \neq i$ scatterers and to the external field [35–37] as:

$$\mathbf{d}_i = \alpha_e \left(\mathbf{E}_{\text{inc}}(\mathbf{r}_i) + \sum_{j \neq i}^N G_{ij} \mathbf{d}_j - \sum_{j \neq i}^N \mathbf{g}_{ij} \times \mathbf{m}_j \right), \quad \mathbf{m}_i = \alpha_m \left(\mathbf{H}_{\text{inc}}(\mathbf{r}_i) + \sum_{j \neq i}^N G_{ij} \mathbf{m}_j + \sum_{j \neq i}^N \mathbf{g}_{ij} \times \mathbf{d}_j \right), \quad (1)$$

where \mathbf{r}_i is the position of the i th dipole, α_e and α_m are electric and magnetic dipole polarizabilities of the scatterers, respectively, and \times denotes a cross-product. Tensors G_{ij} and \mathbf{g}_{ij} describe the interaction between i th and j th dipoles [35–37]. Depending on a particular case under consideration (atoms [30,38] or NPs [6,7]), the expression for the polarizability has to be chosen with a great caution [33,39,40]. Solution of the linear system of equations (1) for the *infinite* 2D *rectangular* array under *normal* incidence, implies that every scatterer in the array has the same electric, \mathbf{d}_i , and magnetic, \mathbf{m}_i , dipole moments:

$$d_x = \tilde{\alpha}_e E_{0x} = \frac{E_{0x}}{1/\alpha_e - S_{xx}}, \quad m_y = \tilde{\alpha}_m H_{0y} = \frac{H_{0y}}{1/\alpha_m - S_{yy}}, \quad (2)$$

where $S_{xx,yy}$ are diagonal elements of 3×3 tensor $S = \sum_{j=2}^{\infty} G_{1j}$ ($i = 1$ dipole is assumed to be located at the center of the array), and $\tilde{\alpha}_{e,m}$ are *effective* electric and magnetic polarizabilities. The tensor S which depends on the in-plane component of the incident wave vector and on the arrangement of scatterers is usually referred as to the *lattice sum*, which describes the interaction between NPs (in this particular case, S is a *dipole sum* [5–7]). Thus the *effective* polarizability $\tilde{\alpha}$ describes the response of a single NP to a plane-wave excitation taking into account entire surrounding of the NP.

The typical behavior of S (Fig. 2(a)) demonstrates the emergence of maxima associated with the *Rayleigh anomalies* [4]. For instance, the constructive interference for the infinite rectangular 2D array located in XOY plane with periods h_x and h_y along X and Y axes occurs if

$$\mathbf{k}_{\parallel} = \mathbf{k}_{\text{inc}} + p\mathbf{K}_x + q\mathbf{K}_y, \quad (3)$$

where $\mathbf{k}_{\parallel} = (k_{\parallel x}, k_{\parallel y})$ is a surface component of an incident wave, $\mathbf{k}_{\text{inc}} = k_{\text{inc}} \hat{\mathbf{k}}_{\text{inc}} = (2\pi n_h / \lambda) \hat{\mathbf{k}}_{\text{inc}}$ are reciprocal lattice vectors of the array, n_h is the refractive index of the host medium; $\mathbf{K}_x = (2\pi/h_x) \hat{x}$ and $\mathbf{K}_y = (2\pi/h_y) \hat{y}$; p and q are integers, which correspond to the orders of diffraction in X and Y directions. For projections on x and y axis, one has $k_{\parallel x} = k_{\text{inc}} \sin \theta_x + 2\pi p/h_x$ and $k_{\parallel y} = k_{\text{inc}} \sin \theta_y + 2\pi q/h_y$, where θ_x and θ_y are angles between Z axis and projections of \mathbf{k} to XOZ and YOZ planes. For example, in a homogeneous environment, the surface component of an incident wave in XOY plane is $k_{\parallel}^2 = k_{\parallel x}^2 + k_{\parallel y}^2$, which, all together, yields the expression $F(\lambda, h_x, h_y, \theta_x, \theta_y) = 0$ defining the RA of the $\{p, q\}$ order. For a given incidence $\{\theta_x, \theta_y\}$ and fixed periods $\{h_x, h_y\}$, F results in the quadratic equation in λ , which for commonly considered normal incidence is solved to be $\lambda = n_h [(p/h_x)^2 + (q/h_y)^2]^{-1/2}$.

Mathematically, the manifestation of CLRs can be understood from the vanishing of the real part of the denominator of Eqs. (2), $\Re[1/\alpha - S] = 0$ (see Fig. 2(a), (g)). These two fundamentally different terms describing the properties of a single particle, $1/\alpha$, and the geometry of the entire structure, S , imply the Fano nature [41] of CLRs representing the coupling between a broad resonance of a single scatterer and a discrete state associated with the light scattered by the array at the wavelength of the RA. The maximum and the dip of Fano-shaped CLRs are also referred as to bright and dark CLRs [42,43], having universal near-field patterns [44,45]. For *oblique* incidence with TM polarization, *out-of-plane* CLRs are found to emerge [46,47]. These CLRs originate from coupling between NPs outside of the plane in which the array of NPs is located. Mathematically, they emerge similarly as in-plane CLRs: $\Re[1/\alpha - S] = 0$, with the only exception that the lattice sum S is different for in-plane and out-of-plane interactions [48]. The elaborative discussion on in-plane and out-of-plane CLRs and necessary conditions for the emergence of the latter are summarized in Refs. [48,49]. In what follows, we will mostly concentrate on the discussion of the *in-plane* CLRs.

CLRs can be observed in regular arrays of any types of scatterers [57]. For most common situations, where ED and/or MD interactions dominate, the only requirement is the vanishing of the real parts of the respective denominator(s) in Eq. (2). Following this requirement, CLRs are demonstrated in Au or Pd thin films with *holes* arrays [58], NPs with dielectric permittivity ϵ such that $\Im(\epsilon) \gg \Re(\epsilon) > 0$ [59] (i.e. supporting localized Zenneck modes [60]), transdimensional nanostructures composed of van der Waals [32,61,62], magnetic [63] or excitonic molecular [64] materials, and atomic arrays [30,65–70]. Moreover, CLRs are

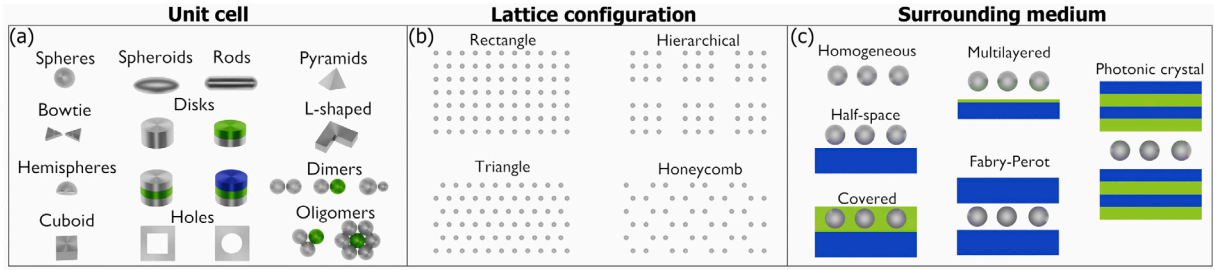


Fig. 1. CLRs are observed in arrays of scatterers with a variety of typical (a) shapes and compositions, specifically arranged with respect to (b) each other and to (c) surrounding medium. Different colors denote different types of composition, which include plasmonic, all-dielectric, magneto-optical materials, atoms, and dyes. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

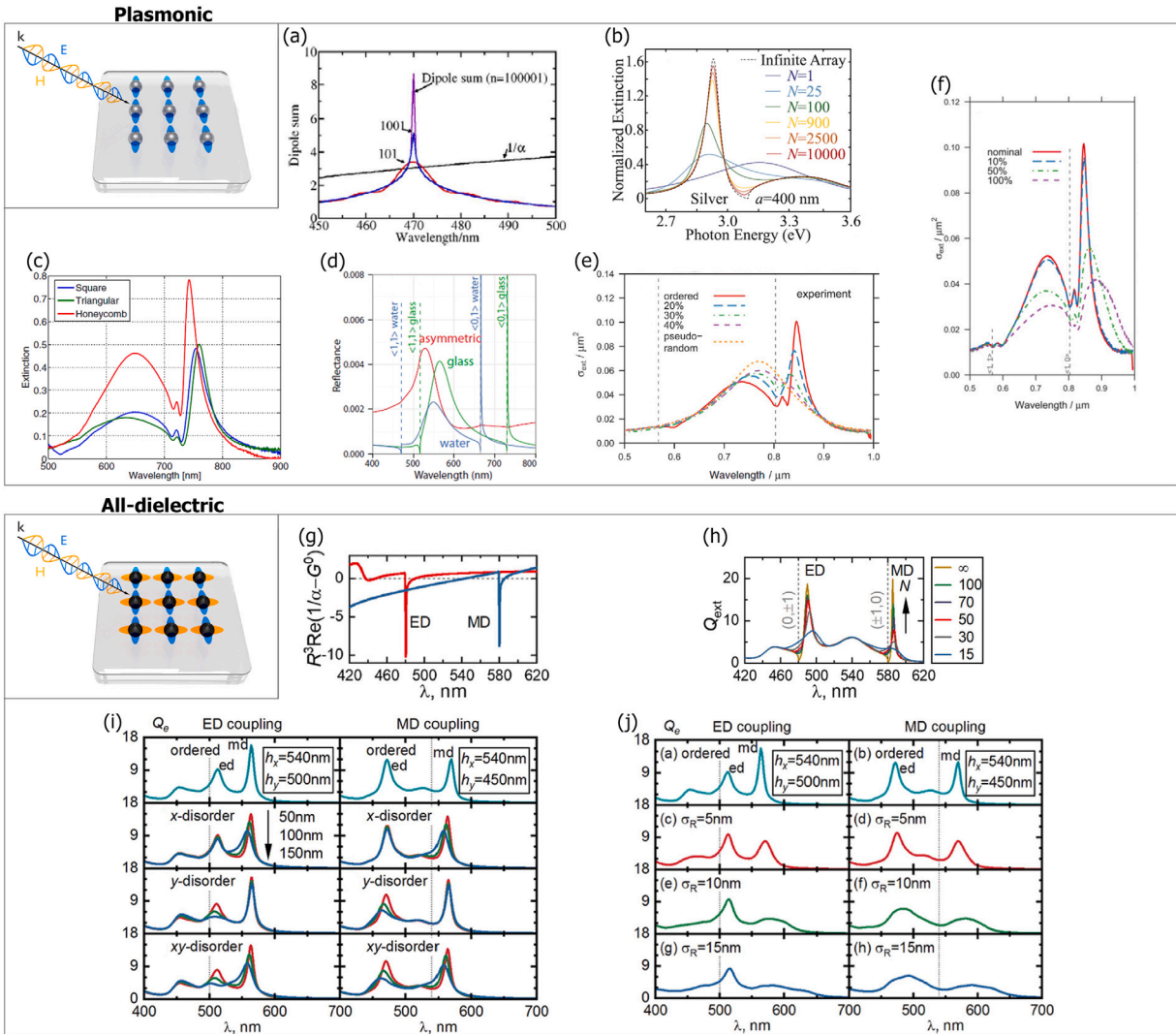


Fig. 2. (a) Real parts of the dipole sum, $\Re(S)$, and of the inverse polarizability, $\Re(1/\alpha)$, for Ag NPs arranged in 1D chain perpendicular to the wavevector direction [50]; (b) extinction for square arrays with different N of constituent Ag NPs [51]; (c) extinction for arrays of Au NPs with different geometrical arrangement [52]; (d) reflectance for arrays of Au NPs in symmetric water, glass, and in asymmetric water/glass environments [53]; extinction for arrays of Au NPs with various degrees of (e) positional and (f) size disorder [54]; (g) normalized denominators of Eqs. (2) for infinite arrays of all-dielectric Si NPs [55] supporting both ED and MD CLRs and (h) respective extinction spectra for finite-size $N \times N$ arrays with different amount of constituent NPs, N . Extinction for arrays of Si NPs with various degrees of (i) positional and (j) size disorder [56]. All figures are adapted with permission. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

not limited to the dipole interaction. Higher-order *multipoles* localized on scatterers can also couple with RAs [71]. Such purely multipole CLRs are of specific interest for mimicking symmetry breaking in symmetric structures to obtain chirality [72] or second harmonic generation [73]. Explicit theoretical solutions for arrays with ED and electric quadrupole (EQ) [74,75], ED, EQ and MD [76–79] and ED, EQ, MD and magnetic quadrupole (MQ) [80] interactions between spherical particles with the extension for semi-analytical approach for cubes and cones [81] have been reported. We notice that most of the CLRs features discussed in the literature are generously studied within the framework of ED or ED+MD approximations, while higher-multipole (EQ, MQ and above) CLRs are still largely unexplored. Interestingly, well-established rigorous treatments, such as T-matrix [82], Korringa–Kohn–Rostoker (KKR) [83,84] and layer KKR (LKRR) [85]¹ methods are usually overlooked in CLRs-related works which involve high-order multipoles, despite the fact that any arbitrary multipole can be self-consistently handled by these methods.

2.2. General properties

Recalling that the *polarizability*, α , of a constituent scatterer is defined by its (i) *shape* and (ii) *composition*, while the *lattice sum*, S , is a function of (i) *geometry* of the lattice (i.e. the arrangement, period and etc.), and (ii) the properties of the *surrounding medium*, it is instructive to consider these fundamental characteristics separately, keeping in mind that both α and S are generally wavelength-dependent. The variation of the angle of incidence provides yet another degree of freedom for tailoring S [88–91], however normal incidence is assumed in most of the works.

Before moving forward, we notice that the role of *disorder* on CLRs in arrays with (i) perturbed size or shape of constituents (which affects polarizability α) and with (ii) perturbed positions of scatterers (which affects the lattice sum S) has been comprehensively addressed for plasmonic [54,92] (Fig. 2(e), (f)), all-dielectric [56] (Fig. 2(i), (j)), and atomic [30] arrays. Generally, CLRs are robust to small fluctuations in either α or S , and rapidly vanish for significant disorder. Moreover, systematic experimental studies revealed that new pseudo-Bragg conditions that do not occur for periodic arrays [93] can emerge in pseudo-random configurations of lattices, while the removal of up to 99% of NPs still conserve the CLRs [94]. The latter is explained by correlated interaction between remaining constituents, thus, interparticle coupling plays a critical role in disordered systems [95,96].

2.2.1. Tailoring CLRs via intrinsic properties of a single scatterer

State of the art fabrication techniques allow for a precise control of shapes [97–101] and sizes of particles from various materials. Engineered in such a way polarizability [102] and eigenmodes [103] of a single scatterer provide a means for tailoring CLRs [104]. Specifically, by appropriately varying the size and material of *plasmonic* NPs, CLRs can be tailored to different spectral ranges, from UV–Vis for Al [99,105–107] to telecom for TiN [108]. As was noticed in 2015 [109], “plasmonic films can easily be better”, i.e. the metal deposition conditions can be easily tailored to get high-quality plasmonic films, which is shown to play a critical role on Q-factor of CLRs in arrays of plasmonic NPs [19,110]. *All-dielectric* building blocks simultaneously supporting strong ED and MD resonances [37], significantly extend the wavelength range of CLRs [111–114]. The important feature of *lossless* dielectric materials is the scalability: the wavelength of CLRs can be varied by simply scaling the whole array [115], which is generally not the case for *lossy* plasmonic materials.

In more sophisticated cases, unit cells may be composed of several NPs (for example, symmetric [116] and asymmetric [117,118] dimers, or stacks [119–121]), which enable polarization-dependent [122,123], chiral [124,125] or toroidal-like [126] responses. CLRs can be also enriched with various unusual properties such as chirality [127], large electric field enhancement [128] and additional multiple resonances [129,130] by mixing different materials within a single unit cell. Additional degrees of freedom for harnessing CLR properties become available by introducing *magnetic* materials, for example, hybrid magneto-plasmonic arrays of Ni and Ag NPs [131] or Ni/SiO₂/Au stacks [132]. *Anisotropic* polarizability (apart from a scalar α assumed in Eqs. (1)) enables bianisotropy of CLRs as shown for nanodisks with eccentric through holes [133].

Once manufactured, the nanostructures are meant to support CLRs within a limited wavelength range, thus the *tunability* is important feature to be implemented. From the perspective of a single scatterer, the most convenient way to achieve real-time tuning of CLRs is to vary the refractive index of NPs, which can be implemented for graphene [51,134] and phase-change materials (for example, Ge₂Sb₂Te₅ [135–137]), or by controlling electrochemical potential (for Au NPs [138]). Alternatively, *magneto-optical* effects comprehensively discussed in Ref. [139] provide a means for tunability by an external field polarization [140,141], by thermoplasmonic heating of Ni NPs to the Curie temperature so that they change their properties through demagnetization or magnetic switching [142], and by externally applied magnetic field which controls optical response of dielectric/magnetic core–shell cylinders [143] or chiral light transmission in Au/Au/Ni trimers [144]. Another way to manipulate optical properties of a unit cell is to induce CLRs in regular arrays of magneto-optical NPs on metal–dielectric substrates, thus a coupling between magneto-optical Ni NPs and Au/SiO₂ substrate can efficiently suppress optical losses in Ni [145].

¹ Respective open-source codes are also available online [86,87].

2.2.2. Tailoring CLRs via lattice configuration and surroundings

Well-established experimental techniques for large-area nanofabrication of regular arrays of NPs (including nanostencil [146] and laser writing lithography [147], DNA-assisted assembly [148,149], and self-assembly [150–153]), provide a means for controllable variations of the lattice *pitch*, which results in a change of the dipole sum S having a peak at period-dependent RA wavelength given implicitly by Eq. (3). On using this knowledge, it becomes apparent how to tailor CLRs through changing the inter-particle distance, which has been extensively discussed for linear 1D chains [5,6,50] and 2D square arrays [14,16,154,155]. *Tunability* from this point of view is implemented in practice via strain [156–158] or mechanical [151,159–161] stretching. Alternatively, S can be modified by *arranging* unit cells in a variety of ways: rings [162], honeycomb [163–166], hexagonal [75,76] and hierarchical [167,168] lattices. Honeycomb lattice represent a special case of interest since this arrangement supports Dirac-like CLRs [169], analogous to graphene. Comprehensive studies contradictory report similar [52] and remarkably different [170–172] properties of CLRs in arrays with square, rectangular, hexagonal, honeycomb, and Lieb arrangements (Fig. 2(c)). By changing the geometry of Bravais lattices [173–175], one can enable or disable a part of the array (depending on polarization of the incident illumination), and tailor CLRs in a broad variety of ways, including engineering super- and subradiant states [176] conveniently described via the hybridization model [177]. Multiple CLR can be constructed on combining several lattices with different unit cells and pitches within a single arrays [178].

While Eqs. (1) describe the interaction of dipoles in a homogeneous environment, in many practical situations scatterers are deposited on a substrate with refractive index different from that of the superstrate, or in a more complex stratified medium. In these cases, tensors G_{ij} and g_j from Eqs. (1) should be modified to take into account reflections from the boundaries between adjacent layers [179–181]. CLRs usually vanish for arrays deposited on a substrate with refractive index different from the upper half-space [53,182] (Fig. 2(d), for purely ED interaction). However, *parallel* CLRs [183–186] (emerging for the incident polarization vector parallel to the diffraction vector participating in the collective mode) may exhibit greater tolerance to the asymmetry of the environment. For higher multipole interactions [77,112,187–191], or for arrays of dimers [192], nanorods [193] or more sophisticated NPs [194], the behavior of CLRs in half-space surrounding is much more involved.

More sophisticated examples of CLRs emerging in arrays located in non-homogeneous environments include NPs embedded *within* [195–198] or *outside* [199–203] of the *Fabry–Pérot* cavities, multilayered thin films [204,205], photonic crystals [197,198], and more complex surroundings [206]. In all of these cases, photonic cavity modes come into play and provide a means for richer hybridization scenarios, which can be conveniently understood from the Fourier modal analysis [207]. Non-homogeneous environment also provides a means for spectral *tuning* of CLRs by using different fluids as a superstrate via microfluidic devices [208], and making use of the temperature dependence of the refractive index of these fluids [150], or by using graphene [209–211] or phase-change materials [212] as an extra-layer.

Although the suppression of CLRs in asymmetric environment is the vital feature for sensing applications discussed below, it is important to mimic the homogeneous environment for preserving high Q-factor of CLRs [99] in most of the other cases. It occurs that quite thin covering layer allows to almost completely mitigate suppression of CLRs [213] or mediate richer hybridization scenarios for thicker layers [214–216]. Another potential route for mimicking homogeneous medium is to put NPs on top of the dielectric nanopillars [217], thus minimizing the coupling between NPs and a substrate. Moreover, the reflective substrate may eliminate the index-matching requirement for vertically aligned antennas [218].

The efficient and convergent calculation of the *infinite* lattice sum S from Eqs. (1) commonly involves the non-trivial Ewald's method well-known for 1D [219] and 2D [220,221] lattices, and summarized in Ref. [222].² Alternative closed-form analytic expressions for 1D [7,225,226] and 2D [227] arrays, the reduction of the lattice sums [228], or the averaging over summations for different number of truncation terms [77] are also extensively discussed in the literature. In reality, however, the array, or its illuminated area have a *finite* size. The impact of the finite size of arrays on CLRs has been comprehensively discussed for 1D [50,229] and 2D [51,192,230–232] arrays of plasmonic NPs with only ED resonance in terms of the extinction spectra, electric field localization [233], and in conjunction with lasing applications [234]. Generally, one may expect the ED CLRs of arrays from $\approx 20 \times 20$ NPs to be almost indistinguishable from the respective infinite lattice (Fig. 2(b)). However, the overall amount of NPs in finite array required to mimic CLRs in infinite lattice depends on size of constituent NPs [233,234] and may vary from case to case. For all-dielectric arrays of NPs with ED and MD CLRs, the magnetic counterpart does not generally approach infinite-lattice approximation [55,91] (Fig. 2(h)) because of strong mutual interaction between ED and MD moments in finite-size lattices. In both ED (plasmonic) and ED+MD (all-dielectric) cases, the dipole moments on NPs located close to the boundaries of the array are generally different from NPs located near the center of the array. EDs and MDs on the latter NPs are well described by Eqs. (2) [51,55].

3. Applications

In what follows, we will make a straightforward connection between fundamental properties of CLRs and their most representative applications, from pioneering works to recent advances.

3.1. Sensing and spectroscopy

The exceptional sensitivity of CLRs to small perturbations in the refractive index of the surrounding medium [53,241] immediately makes arrays exhibiting CLRs perfectly suitable for sensing applications. The concept of CLR-mediated sensing has been

² Respective open-source computer codes are also available online [223,224].

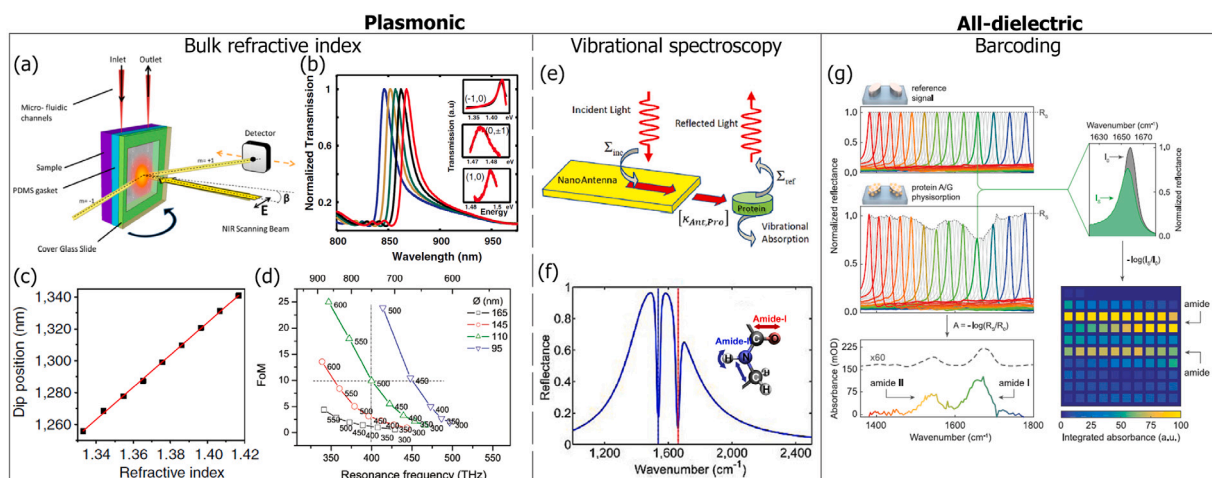


Fig. 3. (a) The example design of the sensor based on arrays of NPs [235]; (b) spectral position of CLRs measured in transmission regime for different values of the refractive index of the sample [236]; (c) typical linear dependence of CLR spectral position on the refractive index of analyte [237]; (d) universal FoM for CLRs-mediated sensing in arrays of Au NPs as a function of their size [238]. CLRs in (e) Au NPs with attached proteins allow to measure (f) narrow dips in reflectance spectra associated with specific vibrational absorption in Amide-I and Amide-II major bands of the proteins [239]. (g) All-dielectric low-loss all-spectral sensing [240]. All figures are adapted with permission.

suggested in 2010 [242,243], though, some earlier works have also discussed the possibility of using arrays of coupled plasmonic NPs for the same purposes [244–246]. The idea is to make use of the non-symmetric surrounding (Fig. 3(a)), and measure spectral shifts of CLRs (Fig. 3(b)) as a function of refractive index of the analyte (Fig. 3(c)). These sensors have extremely large figures of merit (FoM), however, they are only capable of measuring the refractive index, without determining the very nature of the analyte. Nonetheless, it is possible to observe a presence of proteins with naked eyes [236] via CLRs. Moreover, *local* (i.e., in a vicinity to NPs) changes in the refractive index (thoroughly addressed in Ref. [247]) can be measured simultaneously with a *bulk* refractive index by taking the advantage of the Fano shape of CLRs and tracking spectral shifts of both dip and peak of CLRs, respectively [248]. As discussed in Section 2, *polarizability* significantly affects CLRs, thus a lot of studies addressed this feature by estimating sensing performance of NPs with different shapes: mushrooms [237], spheroids [249], disks [250], cubes [251], L-shaped particles [241], bowtie antennas [201,252], dimers [253], rings [254], rods [255], clusters [256], all that with plasmonic Au, with rare exceptions to other metals, such as Al [257,258]. General dependence of FoM for different Au NPs is summarized in Ref. [238] (Fig. 3(d)). Careful manipulations with non-homogeneous or stratified environment may increase sensitivity of CLRs to variations in the refractive index of analyte, as it was shown for Fabry–Pérot cavities [259,260] or planar multilayered structures [201,235,252,258,261,262]. More advanced designs of sensors are based on complex *arrangements* of NPs (self-similar sub-arrays [178] with precise multiresonant sensing in near- and mid-IR regions) or complex *unit cells* (magnetoplasmonics sandwich Ni/SiO₂/Au nanodisks [263]).

Collectively enhanced IR absorption (CEIRA) spectroscopy [239] has a significant advantage since it provides a means for detecting Amide-I and Amide-II bands, thus, unambiguously determining the chemical composition of the analyte (the interested Reader is referred to a comprehensive review [264] on this topic). These CLRs are not necessarily narrow, but spectrally tuned to mid-IR range, which corresponds to fundamental vibrations associated with rotational–vibrational structure of chemicals, thus the dips in the reflection spectra associated with the vibrational absorption of specific molecules can be used to determine a composition of the analyte (Fig. 3(e), (f)). Furthermore, CEIRA enables *real time* biosensing [265], and advanced analysis of the secondary-structural properties [266] and molecular overtones [267]. Recent advances in CEIRA are based on the use of lossless *all-dielectric* metasurfaces, which are easily tuned to exhibit CLRs at a broad variety of wavelengths, including very important mid-IR region. Thus, detailed fingerprints of the analyte can be registered without even using spectrometers (Fig. 3(g)) [240,268,269] by measuring familiar vibrational absorption pixel-by-pixel, where each pixel is tuned to exhibit CLRs at a particular wavelength. Finally, recently reported implementation of CLRs for circular dichroism spectroscopy [270] is another promising application of all-dielectric high refractive index NPs.

3.2. Narrowband transmission, reflection and absorption

Extraordinary light transmission in arrays of holes perforated in Ag film has been experimentally observed back in 1998 [273], with the follow-up theoretical elaboration [274] and observation of similar phenomena for 1D gratings [275]. After some attempts to describe and understand this unusual phenomenon for 1D slits [276,277] and 2D arrays of holes [278–281], a rigorous treatment has been presented by García de Abajo [11,27]. The latter works have rigorously explained extraordinary light transmission via the emergence of CLRs and provided useful guidelines for achieving a complete transmission (Fig. 4(a)) or reflection (Fig. 4(b)) from perfectly conducting thin films with hole arrays or perfectly conducting cylinders, respectively. It is also worth noting, that Refs. [27,279–281] discuss a direct relation of CLRs to Fano-type resonances. The ultra-narrowband *absorption*, being quite

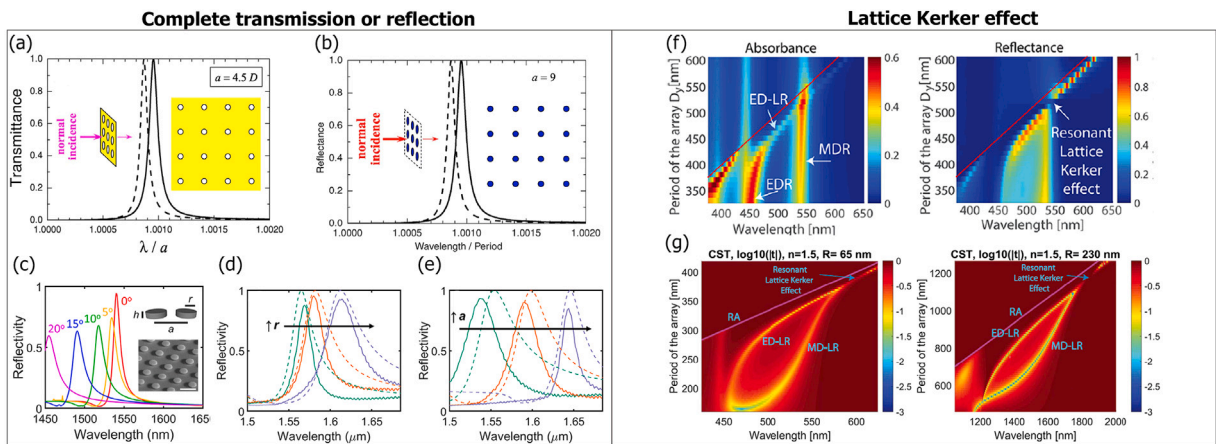


Fig. 4. Exact simulations (solid) and analytical results for full and narrowband (a) *transmittance* [11] and (b) *reflectance* [27] in square arrays of circular holes perforated in a thin perfect-conductor and of perfectly conducting thin circular disks, respectively. Spectral tuning of CLR in a-Si nanodisk arrays [271] by varying (c) angle of incidence, (d) nanodisk radius, and (e) pitch. *Lattice Kerker effect* emerging due to interaction between (f) ED CLRs and single-particle MD [111], and between (g) ED and MD CLRs [272] in (f) rectangular and (g) square arrays of Si NPs. All figures are adapted with permission.

challenging to achieve, has been also experimentally demonstrated for arrays of plasmonic [282] and all-dielectric [283] NPs, again through the emergence of CLR. Yet another interesting phenomenon is so-called electromagnetic induced *transparency* [284–286] which emerges at the wavelength corresponding to the *dip* of the Fano-type CLRs.

All-dielectric NPs are also promising candidates for ultra-narrowband CLR-mediated complete *reflection*. Some preliminary measurements [174] for binary arrays of Si nanodisks have reported quite low reflectivity ($\approx 25\%$), however, general guidelines for optimal design with 100% reflectivity [174] have been further extended and elaborated for Si [287] and for other all-dielectric materials [115,137] with experiments for Si in telecom [271] which confirmed the possibility to reach almost complete (up to $\approx 90\%$) and narrowband reflection. Efficient tuning of the reflection can be achieved by changing the angle of incidence (Fig. 4(c)), size of constituent NPs (Fig. 4(d)) and array pitch (Fig. 4(e)), as it might be obvious from the discussions in Section 2. Simultaneous scaling of Si particles and array period allows to tune the CLR wavelength, in principle, to any arbitrary range, as long as the losses in Si are negligible, for example, to THz regime, as demonstrated for *micron*-sized Si cubes [288]. For specifically designed lattices, the total reflection and total transmission regimes can be switched by simply changing the polarization of the incident illumination [289]. More advanced approaches make use of all-dielectric *magneto-optical* bismuth-substituted yttrium iron garnet NPs [290] for tailoring a complete narrowband transmission accompanied by gigantic Faraday rotation.

Suppression of backscattering initially formulated for a single magnetic sphere with permeability $\mu \neq 1$ by Kerker et al. [291] and revisited for non-magnetic $\mu = 1$ high-index NPs [292–294], is yet another exciting effect which can be engineered in regular arrays of NPs supporting CLRs. *Single* scatterer does not scatter in a backward direction if its electric and magnetic multipoles of the same order and magnitude overlap in-phase [295]. These conditions are realized easily for lossless NPs [293,294], and non-trivially for lossy counterparts [296,297]. For regular *lattices* of NPs, the concept of the Kerker effect [111]³ is the same as for a single NP with an exception that electric and magnetic multipoles may have different origins: ED of a single NP and MD CLRs [111,299] (Fig. 4(f)), or ED and MD CLRs [272] (Fig. 4(g)). On using this additional degree of freedom, CLR-mediated Kerker effect can be tuned with an exceptional flexibility. Further generalization of the Kerker effect includes incorporation of lossy constituents and higher-order multipolar interactions [77,300], simultaneous suppression of forward- and backward scattering [301] and lattice invisibility (simultaneously realized zero reflection and total transmission) [81].

Thus, by tailoring the interaction between resonances localized in single constituents of the regular arrays, and RAs, it is possible to engineer a complete and narrowband transmission, reflection and absorption, and suppressed forward- and backscattering.

3.3. Structural colors

Among various strategies, CLRs are exciting alternative for structural color generation [309–311]. Regular arrays of NPs deposited on a substrate, may support broad CLRs (unlike ultra-narrow CLRs in a homogeneous environment) in visible, which are then used for color generation with an exceptional nanoscale resolution (Fig. 5).

Controllable laser printing of *plasmonic* NPs [304] with adjustable position of plasmon resonance of a single NP (Fig. 5(d)) enabled color printing (Fig. 5(f)). Similar strategy is reported for hole arrays [302] (Fig. 5(b)) via perfect light absorption, with improved saturation and brightness (Fig. 5(e)). Arrays of multiresonant metal–insulator–metal sandwich nanodisks [130] (Fig. 5(a)) enable

³ CLR-mediated and almost completely suppressed reflection in arrays of Si NPs has been experimentally measured earlier [298], without an explicit reference to the Kerker effect.

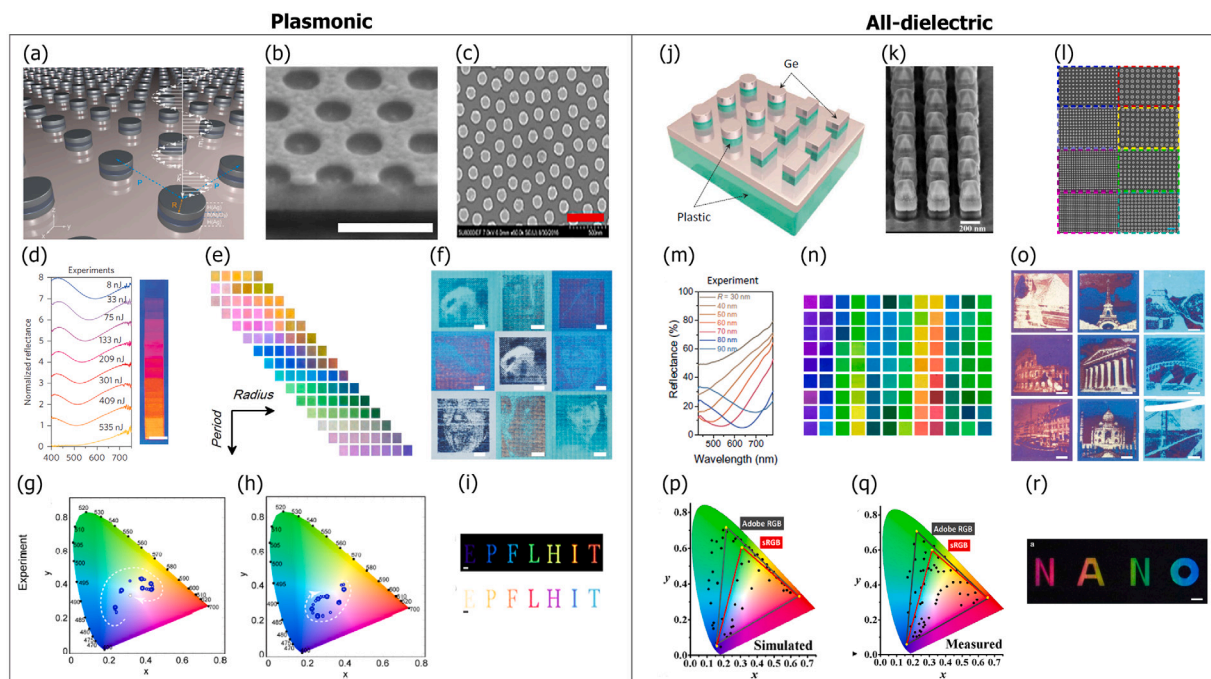


Fig. 5. (a) Schematics of the *plasmonic* metal–insulator–metal nanodisk array [130], and SEM images of (b) arrays of nanoholes in Ag film [302] and (c) lattice of Ag NPs exhibiting correlated disorder [303], all used for color printing. (d) Spectra of the metasurfaces synthesized via laser exposure of different dosage [304], (e) typical color palettes of plasmonic pixels composed of NPs with different sizes and arranged with different periods [302], and (f) respective typical images [304]. Variability of experimentally measured colors in (g) reflection and (h) transmission regimes for arrays of plasmonic NPs with different radii and periods [130], and (i) respective reflection (top) and transmission (bottom) for fabricated letters [130]. Schematic representations and SEM images of *all-dielectric* (j) Ge [305], (k) $\text{SiO}_2/\text{TiO}_2/\text{Si}_3\text{N}_4$ [306], (l) Si [307] metasurfaces. (m) Experimental reflection spectra for Ge NPs [305], (n) typical color palettes [307], and (o) respective sample images printed via all-dielectric metasurfaces [305]. (p) Simulated and (q) measured spectra of color pixels [306]. (r) Optical image created by gradually varying the size and period of Si_3N_4 color pixels [308]. All figures are adapted with permission. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

full-color generation (Fig. 5(g)–(i)). Continuous *tuning* over the entire visible spectrum can be implemented via elastomeric substrates. By stretching the substrate in any of its two dimensions, the period and, therefore, the scattering wavelength is blue- or red-shifted with respect to the resting structure, which allows for covering the entire visible spectrum [157]. Disordered, yet correlated arrays of Ag NPs (Fig. 5(c)) are shown to be promising for heading displays and augmented reality applications [303]. By making use of sensitivity of CLR to asymmetry of the environment and controlling the composition of the superstrate, it is possible to engineer dynamic plasmonic displays with avenues for applications in advanced optical data storage, security tagging and cryptography [312]. Another route for data encryption is the use of non-symmetric oligomers as unit cells, and thus creating a colorimetric information coding mechanism based on polarization- and angular-dependent response [122].

Much the same as for plasmonic NPs [304], the laser printing technique can be extrapolated for *dielectric* constituents [305] (Fig. 5(j)–(l)) capable of producing non-iridescent structural colors. Compared to plasmon counterparts, color surfaces with high-index dielectrics have a lower reflectance, providing excellent color contrast (Fig. 5(m), (o)). Promising design strategy of multipolar modulated metasurfaces based on stacked NPs has been suggested in Ref. [306] (Fig. 5(k)). Such configuration enables suppression of the multipolar modes at nonresonant wavelength, which results in dramatic enhancement in the monochromaticity of reflection spectra. Theoretical (Fig. 5(p)) and experimental (Fig. 5(q)) data demonstrate the possibility of expanding the color palette to Adobe RGB and beyond. Similar strategy has been employed for silicon nitride Si_3N_4 metasurfaces on quartz substrates [308] (Fig. 5(r)). Finally, Si metasurfaces may provide large-gamut, high-saturation, high-brightness, and high-resolution structural color (Fig. 5(n)) [307], which is close to impossible to achieve with other alternative plasmonic–dielectric approaches.

3.4. Fluorescence enhancement

Fluorescent molecules placed in a proximity of NPs arrays supporting CLRs may exhibit enhanced and directional emission once the spatial overlap and coupling between the optical modes and emitter are realized. This phenomenon has been theoretically predicted [318] and experimentally demonstrated [313] for arrays of plasmonic NPs (Fig. 6(a)–(c)) supporting CLRs. Subsequent studies considered a variety of emitters for CLR-enhanced fluorescence: quantum wells [319] and dots [320–323], semiconductor nanocrystals [324] and polymers [325], hexagonal boron nitride emitters [326], and upconverting rare-earth ions [327,328].

General discussion of coupling mechanisms [329–331], Bose–Einstein condensation [332,333], and other systematic studies [334–338] resulted in comprehensive understanding of CLR-enhanced fluorescence. In practice, CLR-enhanced fluorescence is used in organic light emitting diodes [339], to reduce optical etendue [340], for temperature sensing [341], super-resolution mapping of local density of states [342,343], and for coherent control of absorption and modulation of light emission in ensemble of dye molecules [344]. These applications make use of Au NPs since it is a perfect platform for harnessing CLR in the visible range, though alternative materials are also considered, for example, thermally and chemically stable ZrN [345].

Recent progress in all-dielectric photonics provided a means for systematic comparative studies between plasmonic CLR and their all-dielectric counterparts [346,347] in conjunction with fluorescence enhancement, which finally yielded in demonstration of exceptionally narrowband fluorescence enhancement via CLR in lossless Si NPs [314,348] (Fig. 6(d), (e)).

3.5. Lasing

Arrays of NPs exhibiting CLR and incorporated with the gain medium give rise to lasing under strong pumps, which has been demonstrated in 2013 for arrays of nanoholes in Au films [315] (Fig. 6(f)) and Au NPs [316] (Fig. 6(g)). The underlying idea is similar to CLR-enhanced fluorescence: a gain medium is incorporated in the vicinity of plasmonic array, and the frequency of CLR is matched to the lasing transition (Fig. 6(h)). Large field enhancement and increased local density of states [316] near the metal surface yield in enhanced spontaneous emission rate, thus an intense and spectrally narrow peak with a clear threshold as a function of increasing pump power is observed [315]. Comprehensive theoretical models for description of the lasing action in arrays of coupled plasmonic NPs are reported in Refs. [349–351]. In past several years, research has been concentrated on more detailed understanding of ultrafast dynamics preceding lasing action [352], energy-transfer dynamics between plasmonic NPs and dyes [353], modification of band structure [354] and further systematic analysis [355]. Optomechanical processes emerging at ultrafast time scales are shown to modulate CLR inevitably [356]. Exceptionally large modulation bandwidth [357], lasing at K points [164], and in dark and bright modes [358], unidirectionality [359], dual- [360] and multi-modal [361] lasing via engineering *surrounding* and *arrangement* of NPs, respectively, comprise state-of-the-art understanding of CLR-assisted lasing. As for geometrical arrangement, lasing in 1D [362], disordered [94], finite-sized [234] (Fig. 6(k)), quasi-periodic and aperiodic arrays [93] is shown to emerge. Recently, lasing has been also demonstrated for plasmonic arrays in conjunction with colloidal upconverting NPs which provide ultra-stable output at visible frequencies [363], with quantum dots [360,364,365] for tailored polarization patterns and direction of the lasing, and with photochromic molecules for all-optical control over gain and absorption [366]. Real-time *tuning* of the lasing wavelength can be implemented via microfluidic devices [208], stretchable substrates [158,367] (Fig. 6(i), (j)), and

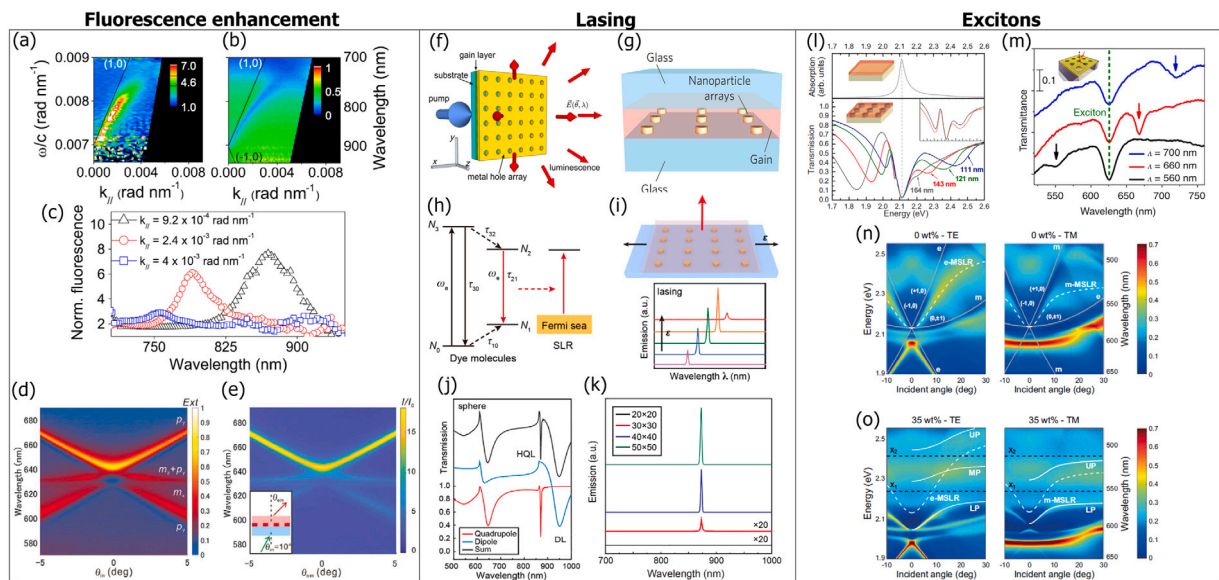


Fig. 6. (a) Fluorescence of dye molecules in the proximity of Au NPs normalized to the fluorescence of dye molecules without Au NPs; (b) transmission of respective arrays of NPs, without dye molecules; (c) fluorescence enhancement spectra at three different k_{\parallel} from (a) [313]. (d) Extinction and (e) photoluminescence enhancement as a function of θ_{in} and θ_{out} , respectively (see inset in (e)), for the Si NP array embedded in a polymer layer containing 3 wt% light emitting molecules [314]. CLR-based lasing in arrays of (f) nanoholes in metal film [315] and (g) Au NPs [316]. (h) Sketch of energy transfer from dye molecules to CLR mode [29]. Recent advances in lasing include (i) active tuning via stretchable arrays [158], (j) exceptionally high-Q lasing with hybrid multipolar ED+EQ CLR [158], and (k) lasing in finite-size [234] arrays. Strong coupling between excitons of TDBC (organic semiconductor) and CLR is observed from (l) absorption (top) and transmission (bottom) of TDBC without (top) and with (bottom) arrays of Ag NPs of different sizes [317]. (m) Excitonic CLR in WS_2 membranes for different array periods [32]. Angle-resolved extinction spectra of the array of all-dielectric Si NPs (n) covered by an undoped PMMA layer and (o) of the same array covered by a PMMA layer doped with dye molecules at 35 wt. % concentration, both for TE and TM polarizations. All figures are adapted with permission.

polarization-dependent response of crescent-shaped [368] or rhombohedral NPs [369]. It is worth to note that high-Q pure EQ [368] or hybrid ED-EQ CLR [158,367], apart from conventional ED CLR are demonstrated for lasing applications.

The supremacy of *plasmonics* for nanolasing [29,370–372] only recently relaxed after the demonstration of large electric field enhancements in lossless *all-dielectric* structures via CLR [373,374] which immediately made them suitable for lasing [375]. Moreover, tunable CLR-mediated multimode lasing at visible wavelengths has been shown in arrays of *ferromagnetic* Ni nanodisks [376].

3.6. Strong coupling

Specific interest is associated with a *strong coupling* between CLR and two-level or general multilayered systems leading to a formation of *exciton-polaritons* [317,377] (Fig. 6(l)) in dyes, which is recently shown to give rise to low-threshold CLR-exciton lasing [378]. For instance, CLR in *plasmonic* arrays are shown to couple with excitons in Rhodamine 6G [379,380], fluorescent organic DiD [381], cyanine dye [382], organic dyes [383,384], quantum dots [385,386], Zn-porphyrin [387], WS₂ mono- [388,389] and multilayers [389], MoS₂ monolayers [390,391], and with anisotropic tetracene crystals [392,393]. In most of these works, Ag is the material of choice for plasmonic NPs, with rare exceptions to Al [382] and Au [388]. All-dielectric Si NPs have been recently proposed for these purposes in conjunction with semiconductors [394] and organic molecules [395] (Fig. 6(n)–(o)). Finally, the hybridization between excitons and RAs (without a presence of any plasmonic or all-dielectric NPs) has been demonstrated recently [32] (Fig. 6(m)), which allows for a manipulation of the visible light at the atomic scale.

3.7. Nonlinear optics

CLR are widely used for enhancing nonlinear processes. In pioneering works [396], it was demonstrated that the second-order nonlinear optical response of noncentrosymmetric metal L-shaped NPs (Fig. 7(a), (b)) can be enhanced by CLR tuned to fundamental wavelength. In the case of nonlinear processes, CLR emerge from the hybridization with *nonlinear* RAs [400,401], which for three-wave mixing read as (cf. Eq. (3)):

$$\mathbf{k}_{\parallel}(\omega_3) = \mathbf{k}_{\text{inc}}(\omega_1) + \mathbf{k}_{\text{inc}}(\omega_2) + p\mathbf{K}_x + q\mathbf{K}_y, \quad (4)$$

where for a particular case of SHG, $\omega_3 = 2\omega_1 = 2\omega_2$.

Routes for CLR-enhanced second-harmonic generation (SHG) include (i) mixing noncentrosymmetric and centrosymmetric NPs within array [402], where centrosymmetric NPs modify the electromagnetic modes of the structure in such a way that SHG from the active particles is enhanced; (ii) adjustment of the incident angle (Fig. 7(c)) [403], including angle-dependent strong circular dichroism [398] (Fig. 7(d)); (iii) controlling external DC magnetic field [404]; (iv) tailoring metasurfaces to support CLR simultaneously at ω and 2ω [405]. Remarkably, fundamental requirement for using noncentrosymmetric systems can be alleviated by breaking the symmetry with *multipolar* CLR [73,397] (Fig. 7(e)) or by significantly enhanced field within the NPs (with MD CLR [406]), both in arrays of centrosymmetric NPs. Regular arrays of plasmonic *hollow nanobumps* are shown to be promising for SHG via CLR due to high damage threshold compared to common isolated plasmonic nanostructures and inexpensive fabrication via direct laser printing [407]. Finally, third-harmonic generation (THG) can also be enhanced via CLR, either directly [408,409], or through cascaded processes [399] (Fig. 7(f)).

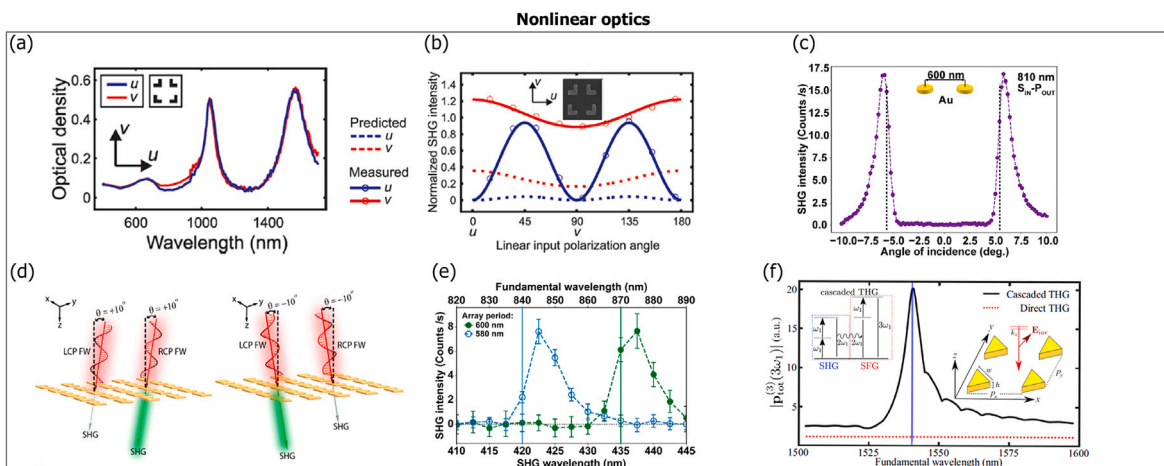


Fig. 7. (a) Extinction spectra of L-shaped Au NPs array (inset shows the unit cell of the sample) and (b) polarization-dependent [396] and (c) incidence-dependent [397] normalized SHG intensity. (d) Schematic of the nonlinear measurement the strong circular dichroism for SHG for illumination with $\pm 10^\circ$ oblique incident with left- and right-circular polarizations fundamental waves on the split-ring resonators metasurface [398]. (e) SHG enhancement via EQ CLR [397]. (f) Cascaded and direct THG [399], the former is enhanced via CLR tuned to 2ω (see inset). All figures are adapted with permission.

4. Discussion and conclusion

The electromagnetic properties of regular arrays of plasmonic, all-dielectric and other types of constituents have been considered in a countless number of publications. In this review, we have discussed the most representative studies which are directly related to *collective lattice resonances*, one of the rapidly developing and exciting phenomenon in electromagnetic light scattering, which in principle can emerge in regular arrays of any types of scatterers once their individual and collective response satisfy resonant conditions outlined in Section 2.

The most promising applications of narrowband CLRs considered here are lasing, fluorescence enhancement, color printing, sensing, and nonlinear optics. It is insightful to notice that general features of magneto-optical activity, multipolar resonances, strong coupling, being translated from one application to the other, provide a solid ground for a plethora of functionalities and promising implementations of CLRs. We notice a clear recent trend of the gradual transition from lossy plasmonic constituents (supporting only ED resonances in most of the cases) to lossless all-dielectric counterparts (with at least ED and MD responses), which is beneficial in most of the cases. Additional multipolar resonances of scatterers significantly enrich CLRs properties, thus, it is expected to observe an upcoming paradigm shift to multipolar CLRs. We also anticipate an increased interest in CLRs emerged in atomically thin structures or CLRs in a strong coupling regime, which allows light manipulation in unprecedentedly thin scale. The *inverse design* [107] along with the deep learning [410] is yet another promising direction, which may be very helpful for finding optimal configurations of structures supporting CLRs.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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